Oscillations

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. Bungee jumping is an example of simple harmonic motion. The long elastic rubber is tied to the ankle of the person who then jumps off from the bridge or a certain height. The jumper is oscillating down and up and undergoes Simple Harmonic Motion due to the elasticity of the bungee cord, albeit at decreasing altitude. When the elasticity comes to rest, the jumper starts swinging like a pendulum till he is pulled back up.

(A) For the problem of the block acted on by spring described by the equation, $a = -\frac{k}{m}x$, find the position, where the acceleration is smallest and also shows that no matter which way the block is moving, it will eventually turn around and move the other way.

(B) What is the minimum condition for a system to execute S.H.M?

(C) Mention the types of oscillatory motion.

Ans.

(A) From equation $a = -\frac{k}{m}x$, a vanishes, when

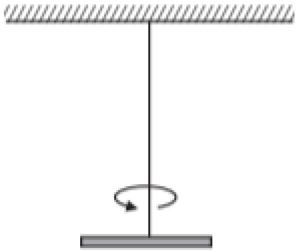
x = 0 that is the unstretched position of the spring and if the block is moving to the right beyond the unstretched position, it will experience an acceleration that will slow it down. Furthermore, the strength of that acceleration will increase as x increases, confirming that it will eventually be strong enough to reverse the direction of the velocity. The same reasoning applies when the block is moving to the left past the unstretched position. In that case, the acceleration is to the right and the block will again reverse its direction of motion.

(B) The minimum condition for a body to possess S.H.M. is that it must have elasticity and inertia.

(C) There are mainly two types of oscillatory motion i.e., linear and circular oscillatory motion. Linear Oscillatory Motion: The object goes left and right or up and down in linear oscillatory motion. For example the vibration of strings of musical instruments. Circular Oscillatory Motion: In Circular Oscillatory Motion, the object moves left to right in a circular motion. For example, the motion of a solid sphere in a half, hollow sphere.

2. A torsion pendulum is analogous to a mass- spring oscillator. Instead of a mass at theend of a helical spring, which oscillates back and forth along a straight line, however, it has a mass at the end of a torsion wire, which rotates back and forth. To set the mass spring in motion, you displace the mass from its equilibrium position by moving it in a straight line and then releasing it. The helical spring (or gravity, depending on whether or not the system is oriented vertically, and in which direction you displace the mass) exerts a (linear) force to restore the mass to its equilibrium position. To set the torsion pendulum oscillating, you turn the mass (rotate it about its center), and then release it. To do this, you must exert torque about the bottom of the torsion wire. The torsion wire, in turn, exerts a restoring torque to bring the mass back to its original position.

(A) If a bar, at the end of a torsion wire, is twisted through some angle and then released, what is the relationship between the angular acceleration α and angular displacement 0?



(a) $\alpha = -\frac{\delta}{I}\theta$ (b) $\alpha = \frac{\delta}{I}\theta$

(c) $\alpha = \delta\theta$ (d) $\alpha = I\theta$

(B) A body executes simple harmonic motion of amplitude 1 cm and frequency 12 cycles per second, then at 0.5 cm displacement, velocity will be:

(a) 60.2 cm/sec

(c) 83.2 cm/sec

(b) 65.3 cm/sec

(d) 77.3 cm/sec

(C) Assume for the above-mentioned figure in (A) the torsion constant is,

8 = 1000 N m/rad and the moment of inertia, = 0.500 kg m2. The bar is rotated through an angle of rad and released from rest, then its period and amplitude of motion is:

(a) 0.140 s and 3.14 rad

(b) 1.042 s and 2.14 rad

(c) 1.042 s and 3.14 rad

(d) 0.140 s and 2.14 rad

(D) The time period of oscillations of a torsional pendulum, if the wire's

torsional constant is, K = 152 J/rad. The rigid body's moment of inertia is 5 kgm.2 about the rotational axis is:

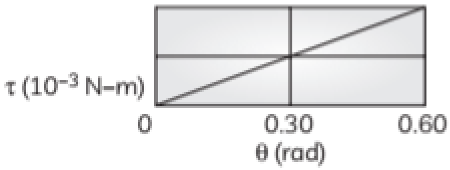
(a) 2 s

(b) 1.04 s

(c) 1.154 s

(d) 3 s

(E) Torsion pendulums are made of a metal disc with a soldered wire running through the centre. The wire is clamped vertically and pulled tautly. The magnitude t of the torque required to rotate the disc about its centre (and thus twist the wire) versus the rotation angle is shown in the figure. The vertical axis scale is defined as, t^2 = 6 x 10-3 Nm. The disc is rotated to a value of, 0 = 0.300 rad before being released.



(a) 8.11×10^{-5} kgm^2
(b) 7.11×10^{-5} kgm^2
(c) 3.21×10^{-5} kgm^2
(d) 8.0×10^{-5} kgm^2

Ans. (A)

(a) $\alpha = -\frac{\delta}{I}\theta$

Explanation: By Newton's third law, the torque Tw exerted by the wire on the external system is equal and opposite to the torque t exerted system on the wire, Tw=-8θ.

As,

T=-8θ

where, & is the torsion constant and t = Ia

where, I is the moment of inertia of the rod about an axis along the wire

Explanation: By Newton's third law, the torque Tw exerted by the wire on the external system is equal and opposite to the torque t exerted system on the wire, Tw=-8θ.

As,

T=-8θ

where, & is the torsion constant and t = Ia

where, I is the moment of inertia of the rod about an axis along the wire

so $I\alpha = -\delta\theta$

or $\alpha = -\frac{\delta}{I}\theta$

(B) (b) 65.3 cm/sec

Explanation: In SHM, Displacement is given by,

$$x = A \sin(\omega t + \phi)$$

$$\frac{dx}{dt} = A\omega\sqrt{[1 - \sin^2(\omega t + f)]}$$

$$= \omega\sqrt{(a^2 - x^2)}$$

$$= 2\pi\sqrt{(a^2 - x^2)}$$

$$= 65.3 \text{ cm/sec}$$

(C) (a) 0.140 s and 3.14 rad

Explanation: As

$$T = 2\pi\left(\frac{I}{\delta}\right)^{\frac{1}{2}}$$

$$= 2\pi\left(\frac{0.500 \text{ kg.m}^2}{1000 \text{ Nm/rad}}\right)^{\frac{1}{2}}$$

$$= 0.140 \text{ s}$$

Since, the bar is released from, θ = л at rest this must be the maximum angle and θA = 3.14 rad.

(D) (c) 1.154 s

Explanation: The time period of oscillation of the torsional pendulum is given by,

$$T = 2\pi\sqrt{\frac{I}{K}}$$

where I is the moment of inertia and K is the torsional constant

$$T = 2\pi\sqrt{\frac{5}{15\pi^2}} = 1.154 \text{ s}$$

(E) (a) 8.11×10^{-5} kg m^2

Explanation: The graphs suggest that

$$T = 0.40 \text{ s and } K = \frac{6}{0.3}$$

$$= 0.02 \text{ N.m/rad.}$$

With these values, Term $2\pi\sqrt{\frac{I}{K}}$ can be used

to determine the rotational inertia:

$$I = \frac{KT^2}{4\pi^2} = \frac{0.02 \times (0.40)^2}{4 \times (3.14)^2}$$

$$= 8.11 \times 10^{-5} \text{ kg m}^2$$

3. If the bob of a vibrating simple pendulum is made of ice, then the period of swing of the simple pendulum will remain unchanged till the location of centre of gravity of the ice bob left after melting the ice remains at a fixed
distance from the point of suspension. If the centre of a gravity of an ice bob after melting is raised upwards, then the effective length of a pendulum decreases and start time period of the swing decreases. If the centre of gravity shifts on the lower side, the time period of the swing increases.

(A) What will happen to the time in a pendulum clock at hills or inside the mines?

(B) Length of a simple pendulum is infinite then why is time not infinite?

(C) If the mass of the pendulum is increased two-fold, then what will be the effect on the periodic time of the pendulum?

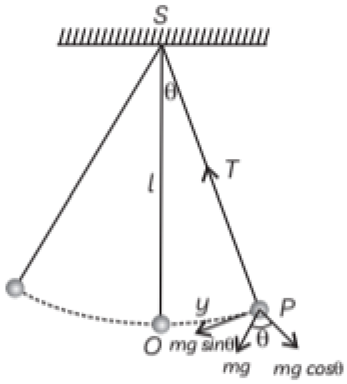
Ans. (A) As time is inversely proportional to the square root of gravity, time period will decrease with the value of g. The value of acceleration due to gravity is less at hills or in the mines than that of on the surface of Earth, the time period of simple pendulum increases at hills or inside the mines. Hence, the pendulum clock will be slowed down which means it will be losing time.

(B) The relation of time period for a simple pendulum is not valid if the effective length of a simple pendulum is more than the
radius of the earth. This means that, T increases up to a certain limit only.

(C) There will be no change in the periodic time because the periodic time is independent of the mass but depends upon the length of the Pentagon and the acceleration due to gravity, which will not be affected if the mass of the pendulum is increased.

4. An ideal simple pendulum consists of a heavy point mass body (bob) suspended by a weightless, inextensible and perfectly flexible string from a rigid support about which it is free to oscillate. But in reality, neither point mass nor weightless string exists, so we can

never construct a simple pendulum strictly according to the definition.
Suppose a simple pendulum of length l is displaced through a small angle from it's mean (vertical) position. Consider m as of the bob is m and linear displacement from mean position is x.



(A) The period of a simple pendulum is doubled, when:
(a) its length is doubled
(b) the mass of the bob is doubled
(c) its length is made four times
(d) the mass of the bob and the length of the pendulum are doubled

(B) The period of oscillation of a simple pendulum of constant length at earth surface is T. Its period inside a mine is:
(a) greater than T
(b) less than T
(c) equal to T
(d) cannot be compared

(C) A pendulum suspended from the ceiling of a train has a period T, when the train is at rest. When the train is accelerating with a uniform acceleration a, the period of oscillation will:
(a) increase
(b) decrease
(c) remain unaffected
(d) become infinite

(D) Which of the following statements is not true? In the case of a simple pendulum for small amplitudes the period of oscillation is:
(a) directly proportional to square root of the length of the pendulum
(b) inversely proportional to the square root of the acceleration due to gravity

(c) dependent on the mass, size and material of the bob
(d) independent of the amplitude
(E) Assertion (A): The frequency of a second pendulum in an elevator moving up with an acceleration half the acceleration due to gravity is 0.612 Hz. not
Reason (R): The frequency of a second pendulum does depend upon acceleration due to gravity.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

Ans. (A) (c) Its length is made four times
Explanation: For a simple pendulum the time period of swing of a pendulum depends on the length of the string and acceleration due to gravity.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

When its length is made four times, then time period of simple pendulum is

$$T_1 = 2\pi\sqrt{\frac{4l}{g}} = 2\left(2\pi\sqrt{\frac{l}{g}}\right) = 2T$$

The period of a simple pendulum is doubled, when its length is made four times.
(B) (a) greater than T
Explanation: Value of g decreases on going below the earth's surface. The time period (T) of a simple pendulum of length, I and acceleration due to gravity g is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{...(i)}$$

$$\Rightarrow \quad T = \frac{l}{\sqrt{g}}$$

Let mine be at a depth h below the surface of the earth having radius R, then

$$g' = g\left(1 - \frac{h}{g}\right)$$

Hence, g decreases.
Therefore, from eqn. (i), T increases.
(C) (b) decrease
Explanation: The pendulum is forced in the opposite direction of the train's motion

while the train is going with acceleration. As a result, the pendulum's effective acceleration will rise. Due to the fact that the time period is inversely related to the square root of the effective acceleration, the time period will shorten.

(D) (c) dependent on the mass, size and material of the bob

Explanation: It is abundantly obvious from the above mentioned equation that a simple pendulum's period is exactly proportional to its square root length and inversely proportional to its square root acceleration owing to gravity. As a result, option (a) and (b) are right. As long as a simple pendulum's motion is simple harmonic, its period is independent of its amplitude. However, it is too large, sin 0, then the motion will oscillate instead of being simple harmonic. Therefore, option (d) is right. It is abundantly obvious from the afore- mentioned equation that the time period of a basic pendulum is independent of the mass of the bob. Option (c) is thus, erroneous.

(E) (c) A is true but R is false.

Explanation: Frequency of second pendulum n = (1/2)s-1. When elevator is

moving upwards with acceleration $\frac{g}{2}$, the effective acceleration due to gravity is:

$$g = g + a = g + \frac{g}{2} = \frac{3g}{2}$$

As $$n = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \text{ so } n^2 \propto g.$$

$\therefore$

$$\frac{n_1^2}{n_2^2} = \frac{g_1}{g} = \frac{3g/2}{g} = \frac{3}{2}$$

or $$\frac{n_1}{n} = \sqrt{\frac{3}{2}} = 1.225$$

or, $$n_1 = 1.225n$$

$$= 1.225 \times \left(\frac{1}{2}\right) = 0.612 \text{ s}^{-1}.$$